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Jet Physics at LEP

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Jet physics at LEP

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The results of studies of the jet structure of hadronic Z^0 decays performed in the first year of LEP operation are reviewed. The measurements of the QCD coupling constant $\alpha_s(M_Z)$ and the detection of the presence of the triple gluon vertex are summarized. After a brief review of the promising status of QCD in relation to even the very soft processes, the running of the coupling constants to high energy is considered in the context of grand unified theories. The necessity and importance of further theoretical work is stressed.

1. Introduction

About 85% of all visible decays of the Z^0 are hadronic. The visible secondaries in a typical hadronic decay consist of some 20 strongly interacting charged particles (mostly $\pi^{\pm}$, some $K^{\pm}$, p, $\bar{p}$, etc.) plus some 20 neutrals (mostly γ from $\pi^0 \rightarrow 2\gamma$ decay, also K^0, n, $\bar{n}$, etc.). The challenges these events present are (1) to understand their properties in terms of quantum chromodynamics (QCD) – our theory of strong interactions in terms of the underlying quark and gluon interactions, (2) to use the data to test QCD as rigorously as possible, and (3) to search these events for new phenomena (new particles, new dynamics).

QCD is a gauge theory, i.e. it is fully defined by the principle that the physics is invariant under a local gauge variation (plus a few numbers determined by experiment, to be predicted later by a deeper theory). Einstein's theory of gravity was perhaps the first to be constructed along these lines. Varying the position coordinates of a test particle (i.e. varying the gauge) by arbitrarily different amounts at different places (i.e. locally) introduces accelerations. Keeping the physics invariant under such a transformation requires the introduction of a compensating field. This field can, equivalently, be either a field of geometrical distortions (i.e. euclidian $\rightarrow$ riemannian geometry) or a field of gravitational forces satisfying the principle of equivalence between accelerations and gravitational forces.

QCD is somewhat different since its gauge variation is in a different space. There are three types of charge (conventionally called red, green and blue), because the quark–quark forces saturate when three quarks are present inside a baryon (i.e. proton, neutron, etc.). The local gauge variation is not a transformation of spatial coordinates but is a local transmutation of these colour charges. To be able to effect such transmutations, the compensating field must also carry these colour charges. Since the quanta of the gauge field (the 'gluons') therefore carry these colour charges, they interact directly with each other. In practical terms, this is the main difference between QCD and quantum electrodynamics, QED, where photons in contrast carry no electromagnetic charge, hence do not interact directly with each other.

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Both in QED and in QCD, the effective coupling constant varies with the square of 4-momentum transfer, Q^2. In QED the cloud of virtual photons surrounding a charge produces a cloud of e^+e^- pairs. This cloud is polarized by the initial charge ('vacuum polarization') so the effective charge is reduced at large distances, i.e. at low Q^2. Consequently, although $\alpha = \frac{1}{137}$ for small values of Q^2, $\alpha(Q^2 = M^2_{Z^0}) = \frac{1}{128}$. In QCD, the same effect occurs: a colour charge is surrounded by cloud of gluons which produces a cloud of $q\bar{q}$ pairs which is polarized. But the cloud of gluons also produces further gluons, directly. This tends to increase the effective charge at low Q^2 and outweighs the previous effect. Consequently the QCD coupling constant α_s diverges at low Q^2. This leads to the unobservability of free quarks and gluons because of their confinement at low Q^2, where it also makes calculations far more difficult, and to 'asymptotic freedom' at high Q^2 where perturbation theory can still work.

This implies the following picture of the bulk of the hadronic events observed at LEP: (i) the process starts with $q\bar{q}$ formation; (ii) as the q and $\bar{q}$ separate, some high-Q^2 process may occur ('hard' gluon emission, etc.); (iii) but certainly many lower-Q^2 processes will occur ('soft' parton showering, where 'parton' = quark or gluon); (iv) which will be followed by an 'infinite' number of very soft processes which result in the partons being formed into hadrons, 'hadronization'; (v) and finally any unstable hadrons decay.

This is a very complex chain of processes!

2. Monte Carlo simulation programs

To evaluate the theoretical predictions in detail, the usual procedure is to use Monte Carlo simulation programs to imitate every part of this complex chain and to generate simulated events which can then be compared with real data. The simulation of the hard processes and of the soft parton shower is based on formulae derived in perturbative QCD theory. That of the hadronization process, however, involving very large values of the coupling constant α_s, cannot be treated in this way and models, more or less inspired by QCD, have to be used instead. The decays of unstable hadrons are treated using experimental data.

Experimental detector effects (inefficiencies, measurement errors, etc.) and other complications such as initial state radiation (radiation of real photons from the initial e^+ or e^-) can also be included in the Monte Carlo simulation. The comparison between simulation and real data can be done at the hadron level, after correcting for detector effects and initial state radiation only, or at some parton level, after correcting for the effects of the hadronization process.

JETSET (Sjöstrand 1982, 1983; Sjöstrand & Bengtsson 1987) and HERWIG (Marchesini & Webber 1988) are the Monte Carlo programs most commonly used at LEP. Both use the parton shower (PS) model based on leading log approximation formulae. These cover many orders of the perturbative expansion of QCD (but no loops, only branching processes) and give excellent agreement with all available data. JETSET provides also the possibility of using complete matrix elements (ME), covering the perturbative expansion only up to α_s^2 but including loops. Using these one can test QCD and determine α_s. However, the parameters of the subsequent fragmentation model need to be retuned at each energy.

There are three more or less commonly used fragmentation models. The independent fragmentation (IF) model, in which each parton fragments into hadrons independently of other partons, is the oldest and is available in JETSET, but due to technical problems in its original formulation and the fact that it failed to fit various aspects of the PETRA data, it has not been used much at LEP so far. This may change with the incorporation of recent improvements (Biddulph & Thompson 1989). HERWIG uses the cluster fragmentation (CF) model, in which colourless structureless clusters formed from $q\bar{q}$ pairs at the end of the parton shower decay isotropically according to phase space. JETSET principally uses the string fragmentation (SF) model, in which the breaking of a linear colour dipole field stretched between the initial q and $\bar{q}$ creates further $q\bar{q}$ pairs. Any hard gluons or soft gluons that had been emitted in the perturbative phase of the event generation simply distort the shape of the string.

3. Determinations of the coupling constant $\alpha_s(M_Z)$

It is important to determine $\alpha_s(M_Z)$ both in as many ways as possible, as a test of the consistency of QCD with the data, and as precisely as possible, as a test of theoretical extensions going beyond the Standard Model (see later).

3.1. $\alpha_s(M_Z)$ *from jet rates*

There are various algorithms for reconstructing jets out of the observed hadrons. The most commonly used is the JADE algorithm (Bartel *et al.* 1986) in which: (i) $m_{ij}^2 = 2E_i E_j(1-\cos\theta_{ij})$ is computed for all particle pairs; (ii) if the lowest m_{ij}^2 value is below $y_{cut} E_{cm}^2$ then particles i and j are combined to form a new (pseudo) particle k, $p_k = p_i + p_j$; (iii) this is repeated until m_{lm}^2 exceeds $y_{cut} E_{cm}^2$ for all remaining (pseudo) particle pairs; (iv) the remaining (pseudo) particles are by definition the jets.

Clearly, the same event may be classified, somewhat arbitrarily, as either a 2-jet, 3-jet, 4-jet, or n-jet event, depending on the value of y_{cut} used. Figure 1 shows a typical plot of the number of jets reconstructed using the JADE algorithm as a function of y_{cut}. It is also interesting to observe that although in principle current ME models cannot generate events with more than four jets, since they are limited to α_s^2 processes, when the fragmentation parameters are tuned to give good agreement with the PS model predictions the jet algorithm gives the same jet-number distribution for ME events as for PS events. Tests on Monte Carlo data show that the JADE algorithm gives better agreement between the number of jets reconstructed from the observed hadrons and the number found at the parton level than other algorithms used so far, if only because the $2\rightarrow 3$ jet transitions happen to balance the $3\rightarrow 2$ jet ones (de Boer 1990). However, at the recent Durham workshop (Pennington 1991), a modification $m_{ij}^2 \rightarrow m_{ij}^2 \times \min(E_i/E_j, E_j/E_i)$ was proposed to reduce the probability of small gluons being formed into separate jets.

To $O(\alpha_s^2)$, the relative production rates of 2-, 3- and 4-jet events are given by

$$R_2 = 1 + C_{21}\alpha_s + C_{22}\alpha_s^2, \tag{1}$$

$$R_3 = C_{31}\alpha_s + C_{32}\alpha_s^2, \tag{2}$$

$$R_4 = C_{42}\alpha_s^2, \tag{3}$$

where
$$\alpha_s(\mu^2) = \frac{12\pi}{(33-2n_f)\ln(\mu^2/\Lambda^2)}\left(1 - 6\frac{(153-19n_f)\ln\ln(\mu^2/\Lambda^2)}{(33-2n_f)^2\ln(\mu^2/\Lambda^2)}\right) \tag{4}$$

and C_{22} and C_{32} depend on μ^2 in such a way that R_2 and R_3 are approximately independent of μ^2 while C_{42} does not, so the 4-jet R_4 is very sensitive to μ^2.

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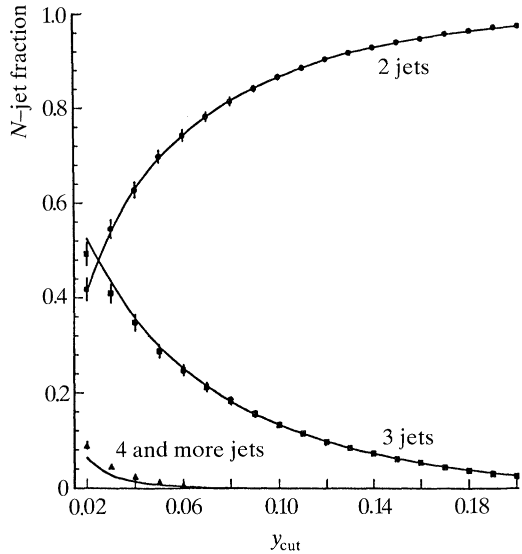

Figure 1. Dependence of the numbers of events classified as 2-, 3- and 4-jet events by the JADE algorithm on the value of the parameter y_{cut}. ●, ■, ▲, Data, corrected for detector effects and photon radiation; ——, QCD + hadronization, $\Lambda = 190$ MeV, $\mu^2 = 0.08$ s.

Table 1. $\alpha_s(M_Z)$ *values obtained from the analysis of jet rates, with their experimental errors*

(In addition there is a theoretical error of about ± 0.008 (or ${}^{+0.007(\mu^2=M_Z^2)}_{-0.011(\mu^2=M_B^2)}$) in the Durham convention (see Pennington 1991) due mainly to scale uncertainties.)

Komamiya *et al.* (1990)	(Mk II)	0.123 ± 0.009
Decamp *et al.* (1991*a*)	ALEPH	0.121 ± 0.004
Abreu *et al.* (1990*b*)	DELPHI	0.114 ± 0.005
Adeva *et al.* (1990*b*)	L3	0.115 ± 0.005
Akrawy *et al.* (1991*b*)	OPAL	0.118 ± 0.003

The parameter μ is the 'renormalization scale' parameter. There are many renormalization schemes in QCD; to $O(\alpha_s^2)$, changes of scheme are equivalent to changes of μ^2. The value of μ^2 is expected to be approximately the same as the effective value of Q^2 in the process, i.e. to be process dependent and lying somewhere between M_Z^2 and about m_B^2. It is an artificial parameter, in the sense that if the calculations could be extended to all orders of perturbation theory the dependence on μ^2 would disappear. Meanwhile, the observable dependence gives one estimate of the magnitude of the uncalculated higher-order corrections.

A typical way to fit the jet rates is to fit the distribution in y_3, the value of y_{cut} where an event changes from the 2-jet to the 3-jet classification, in the range above 0.04 where the 4-jet rate is small. Present results from the various experiments analysing Z^0 decays are very consistent, as shown in table 1, but are limited by theoretical uncertainties.

3.2. $\alpha_s(M_Z)$ *from global event shape variables*

Many variables have been invented to characterize the global shapes of the events (indeed, y_3 could be considered as one of them). For comparison with hard QCD (e.g. ME) calculations, variables should be chosen that are insensitive to the collinear

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Table 2. $\alpha_s(M_Z)$ *values extracted from event shape variables*

(The errors quoted by Magnoli *et al.* include a small-scale uncertainty from $\frac{1}{4}M_Z$ to M_Z while the second error quoted for the ALEPH analysis corresponds to a scale uncertainty from M_B to M_Z.)

variable	OPAL data (Magnoli *et al.* 1990)	ALEPH (Decamp *et al.* 1991*a*)
thrust	$0.132^{+0.017}_{-0.013}$	$0.119 \pm 0.014^{+0.012}_{-0.025}$
C parameter	$0.125^{+0.013}_{-0.011}$	$0.112 \pm 0.017^{+0.011}_{-0.023}$
oblateness	$0.161^{+0.027}_{-0.058}$	$0.186 \pm 0.036^{+0.041}_{-0.002}$

Table 3. $\alpha_s(M_Z)$ *values extracted from pre-clustered event shape variables using* $y_{\text{cut}} = 0.03$

variable	ALEPH: pre-clustered events (Decamp *et al.* 1991*b*)
thrust	0.123 ± 0.004 (expt) ± 0.006 (frag)$^{+0.007}_{-0.012}$ (scale)
C parameter	0.124 ± 0.004 (expt) ± 0.006 (frag)$^{+0.007}_{-0.013}$ (scale)
oblateness	0.115 ± 0.004 (expt) ± 0.005 (frag)$^{+0.006}_{-0.010}$ (scale)

divergence (splitting a momentum into two collinear momenta) and the infrared divergence (adding a zero-energy particle) of QCD. A typical such variable is the 'thrust'

$$T = \max\left(\Sigma_i |p_{\parallel i}| / \Sigma_i |p_i|\right), \tag{5}$$

where the direction that maximizes the thrust value is called the 'thrust axis'. Other such variables in common use include the 'oblateness' and the 'C-parameter' (see Kunszt & Nason (1989) for definitions and discussions). Distributions of the events in such variables have been found to be in good agreement with the predictions of the PS QCD model (Aarnio *et al.* 1990; Akrawy *et al.* 1990*a*; Decamp *et al.* 1990). They also agree well with current ME models after appropriate retuning of their fragmentation parameters (see De Boer *et al.* 1991). However, the values of α_s extracted from the shapes of these distributions (see table 2) have relatively large errors.

The ALEPH collaboration (Decamp *et al.* 1991*b*) have analysed the event shape distributions obtained after first applying the JADE cluster algorithm to the observed hadron system to form the jets. They found the results, shown in table 3, to be independent of y_{cut} for y_{cut} values above 0.03 and also relatively unaffected by fragmentation uncertainties.

However, they point out that in this y_{cut} region there are few 4-jet events and, as configurations of three jets or less that conserve energy and momentum are specifiable by only two variables, that all values of α_s extracted from events with three or fewer jets must be correlated, there being only two independent variables at the jet level.

3.3. *The energy–energy correlation and its asymmetry*

The energy–energy correlation is simply the histogram of the angle χ_{ij} between all pairs of tracks in an event, weighted by $E_i E_j / E^2_{\text{cm}}$, the product of their energies divided by the total energy of the event. This distribution (see figure 2*a*) is strongly peaked near 0° and 180°. The contribution from 2-jet events is symmetric about 90°. However, since a gluon tends to make a small angle with the quark that emitted it, at the jet level a 3-jet event tends to contain one small angle with a low weight and two larger angles with larger weights. Thus gluon emission introduces an asymmetry about 90°.

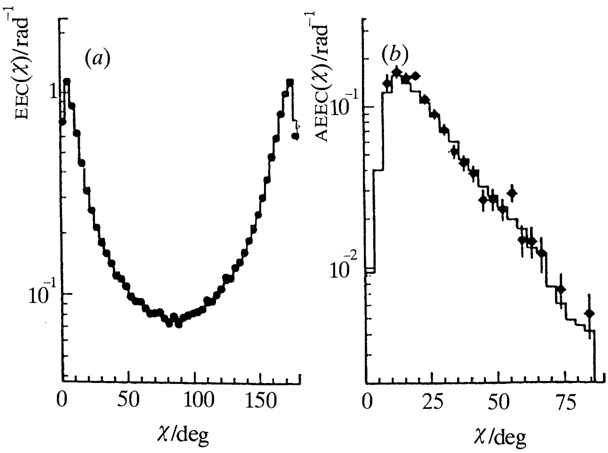

Figure 2. Form of the distribution of (*a*) the energy–energy correlation (EEC) and (*b*) its asymmetry (AEEC). ●, Data; ——, ME option.

Table 4. $\alpha_s(M_Z)$ *values extracted from the energy–energy correlation* (EEC), *its asymmetry* (AEEC), *and the clustered energy–energy correlation* (CEEC) *and the estimated contributions to the error*

(Where the experimental error is not quoted it is included with the fragmentation uncertainty.)

		α_s	(expt)	(frag)	(scale)	(theory)
AEEC						
Abreu *et al.* (1990*c*)	DELPHI	0.106	± 0.003	± 0.003	$^{+0.003}_{-0.000}$	—
Akrawy *et al.* (1990*c*)	OPAL	0.117	—	± 0.007	small	$^{+0.006}_{-0.002}$
Adeva *et al.* (1991)	L3	0.115	± 0.004	$^{+0.007}_{-0.004}$	$^{+0.002}_{-0.000}$	$^{+0.003}_{-0.005}$
EEC						
Akrawy *et al.* (1990*c*)	OPAL	0.117	—	± 0.007	± 0.010	± 0.007
Adeva *et al.* (1991)	L3	0.121	± 0.004	± 0.002	$^{+0.009}_{-0.006}$	± 0.006
CEEC						
Decamp *et al.* (1991*b*)	ALEPH	0.118	± 0.002	± 0.005	$^{+0.006}_{-0.010}$	—

Values of α_s may be extracted either from the energy–energy correlation (EEC) or from its asymmetry (AEEC), shown in figure 2*b*. Normally, one uses only the region away from the forward and backward peaks since these contain no information on α_s but do depend strongly on fragmentation effects. The results are summarized in table 4. ALEPH have used the clustered energy–energy correlation (CEEC) obtained using the jets reconstructed with the JADE clustering algorithm, rather than using the observed particles themselves.

3.4. *Summary of α_s measurements*

The α_s values found by the different methods are in excellent agreement (see table 5). But they are not fully independent, since a 3-jet event can be specified by just two numbers. The scale and 'theory' errors already dominate over the experimental errors everywhere, even though most of the numbers reflect only part of the data taken in LEP's first year of operation. The AEEC method is notable for its very small-scale error. This may be significant, implying that this technique is the most reliable and the least correlated with the rest. But it could be accidental. Further theoretical calculations, both to resolve discrepancies between existing calculations which are

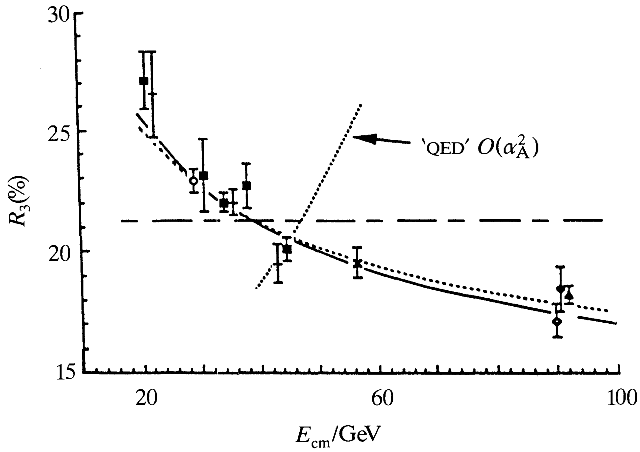

Figure 3. The dependence of the fraction of events classified as 3-jet by the JADE algorithm with $y_{cut} = 0.08$, which is approximately proportional to α_s, on energy which is proportional to Q^2. The fraction falls as the energy rises, in agreement with the expectation from QCD but not with that from a QED-like theory. ▲, OPAL; ◇, DELPHI; ◆, L3; ■, JADE; +, TASSO; ○, Mark II; ×, Tristan (AMY, VENUS, TOPAZ). ——, α_s = const.; -----, QCD $\mu^2 = E^2_{cm}$; $\Lambda_{\overline{MS}} = 250$ MeV; ——, QCD $\mu^2 = 0.0017E^2_{cm}$; $\Lambda_{\overline{MS}} = 107$ MeV.

Table 5. *Summary of $\alpha_s(M_Z)$ values and estimated error contributions*
(Central values have been adjusted to symmetrize the errors.)

	α_s	(expt) × (frag)	(scale)	(theory)
jet rates	0.116	±0.002	±0.009	—
event shapes	0.120	±0.011	±0.018	—
clustered event shapes	0.119	±0.005	±0.009	—
EEC and CEEC	0.118	±0.005	±0.008	±0.007
AEEC	0.114	±0.006	±0.001	±0.004

the main reason for the 'theoretical' errors quoted, and to progress to $O(\alpha_s^3)$ to improve and control the scale uncertainties, are clearly an essential prerequisite if there is to be significant further progress.

4. Search for the triple-gluon vertex

As emphasized in the introduction, the direct coupling between gluons is the crucial feature of QCD. One consequence is the direction in which α_s runs as Q^2 increases. Figure 3 (taken from de Boer 1990), shows that the 3-jet rate, which is approximately proportional to α_s, falls as Q^2 rises, in agreement with QCD and in disagreement with an abelian theory (one with no direct gluon–gluon coupling).

The presence of the gluon–gluon coupling can be demonstrated more directly, using LEP data alone, by analysing the angular distributions of the jets in 4-jet events. There are three categories of 4-jet events: two jets are the q and $\bar{q}$ produced by the Z^0 decay, then either (1) each of them emits a gluon ('double gluon bremsstrahlung'), or only one of them emits a gluon and this turns into either (2) another $q\bar{q}$ pair ('four-quark process') or (3) a gluon pair ('triple-gluon vertex process'). If the jets are ordered in energy, such that $E_1 > E_2 > E_3 > E_4$, the two lowest energy jets are usually the secondary jets.

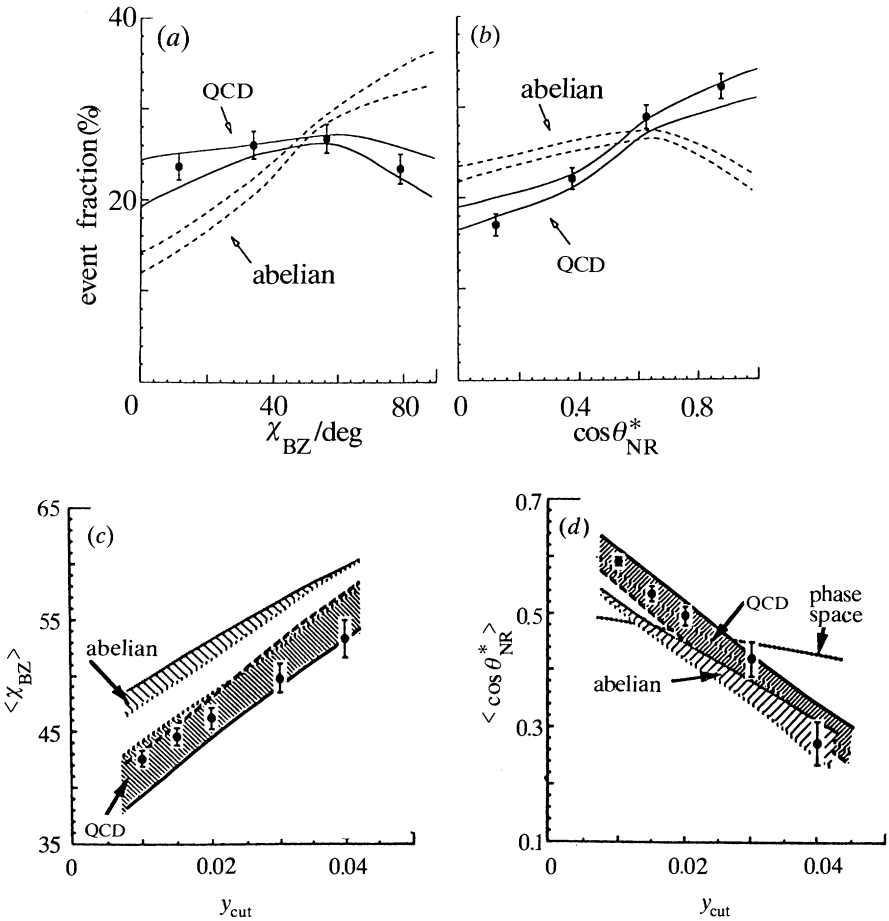

Figure 4. The measured distributions and mean values as a function of y_{cut} of the angle χ_{BZ} and of $\cos\theta^*_{NR}$ agree with the predictions of QCD and disagree with those of an alternative QED-like ('abelian') theory. (c), (d) OPAL: ●, data; ——, QCD shower (JETSET); – – – –, QCD shower (HERWIG); - - - -, QCD $O(\alpha_s^2)$; ——, abelian shower (JETSET); ....., abelian $O(\alpha_s^2)$.

L3 (Adeva *et al.* 1990*a*) and OPAL (Akrawy *et al.* 1991*a*) compared their data directly with the predictions of a QED-like abelian model. In such a model the triple-gluon vertex process would be absent and the ratio of the other two processes would be different. The distributions in (1) the angle χ_{BZ} between the plane containing the jet momenta p_1 and p_2 and that containing p_3 and p_4 and in (2) the cosine of the angle θ^*_{NR} between $p_1 - p_2$ and $p_1 - p_2$ are both sensitive to this change. As can be seen from figure 4, the results agree well with the QCD predictions and disagree strongly with the QED-like alternative.

However, it turns out to be possible to reproduce the QCD predictions for the distributions in χ_{BZ} and θ^*_{NR} even without a triple-gluon vertex contribution, by suitably adjusting the ratio between the other two contributions. Therefore another angle needs to be looked at as well, to discriminate between the double-gluon bremsstrahlung and four-quark contributions. A suitable one is the angle between the two lowest energy jets, α_{34}, because they tend to be parallel in the four-quark case and antiparallel in double-gluon bremsstrahlung. The DELPHI analysis (Abreu *et al.*

1991*a*) fitted the two-dimensional distribution of $\cos\theta^*_{NR}$ and α_{34} to extract the triple-gluon contribution directly. The ratio between the observed triple-gluon contribution and the one predicted by QCD was 1.13 ± 0.24 (stat) ± 0.20 (syst), i.e. 1.13 ± 0.32, in agreement with QCD and almost 4 standard deviations from zero.

5. Particle spectra and multiplicities

In dealing with the soft QCD processes, one key idea is that of local parton hadron duality (LPHD) (see review by Dokshitzer *et al.* 1988) according to which, at asymptotically high energy, hadron distributions should be related to the parton distributions calculated in perturbative QCD by simple scaling factors. Their energy variations should also be closely related since, to first order, any distortions of the distributions by the hadronization process should be energy independent.

A second one is that of soft gluon coherence (see Dokshitzer *et al.* 1988), which is closely related to the Chudakov effect in which a newly converted $\gamma \rightarrow e^+e^-$ pair does not ionize the matter it traverses until the e^+ and e^- separate sufficiently, because destructive interference prevents the emission of soft photons. Similarly, in a parton shower the emission of gluons is suppressed except for gluons emitted very near to their parent's direction. Although Monte Carlo programs are essentially probabilistic, such coherence effects have been incorporated (Marchesini & Webber 1988) by implementing an angular ordering requirement (see Dokshitzer *et al.* 1988).

Combining these two ideas gives the prediction that, at asymptotically high energies, the logarithm of the momentum p of the hadrons from e^+e^- annihilation should have a nearly gaussian distribution whose central value should vary approximately as $E_{cm}^{\frac{2}{3}}$. These predictions were strikingly confirmed in an analysis by the OPAL collaboration (Akrawy *et al.* 1990*b*). Apparently, asymptopia has arrived!

Similarly, it has been demonstrated that the growth with increasing energy of the mean number of charged particles produced in such events follows QCD predictions (Abreu *et al.* 1991*b*) and that the 'KNO scaling' behaviour of the multiplicity distributions (Abreu *et al.* 1991*b*) is demonstrable in a wide class of branching processes including, for example, that embodied in the PS model (Chliapnikov & Tchikilev 1990).

Searches for unexpected non-poissonian spikiness in event structures ('intermittency') have become topical recently. Events with surprisingly spiky rapidity or pseudo-rapidity distributions have been seen both in cosmic rays (Burnett *et al.* 1983) and in accelerator experiments (Adamus *et al.* 1987). An efficient way to search for such effects systematically is by using normalized factorial moments (Bialas & Peschanski 1986, 1988)

$$F_j = \langle (n_m + 1 - 1)(n_m + 1 - 2) \dots (n_m + 1 - j) \rangle / \langle n_m \rangle^j \qquad (6)$$

of a histogram, where n_m is the number of entries in bin m, as a function of the number of bins into which the histogram is divided. Such a plot should be flat if the fluctuations are poissonian, but should rise as

$$F_j = \text{const.} \times 1/\delta y^{f_j} \qquad (7)$$

with $f_j > 0$ in the case of a fractal (self-similar) cascade – until the fractal structure ceases or is lost in the experimental resolution.

The first analysis of e^+e^- annihilation data from PETRA (Braunschweig *et al.* 1989) indicated strong fractal (intermittent) behaviour greatly in excess of the predictions

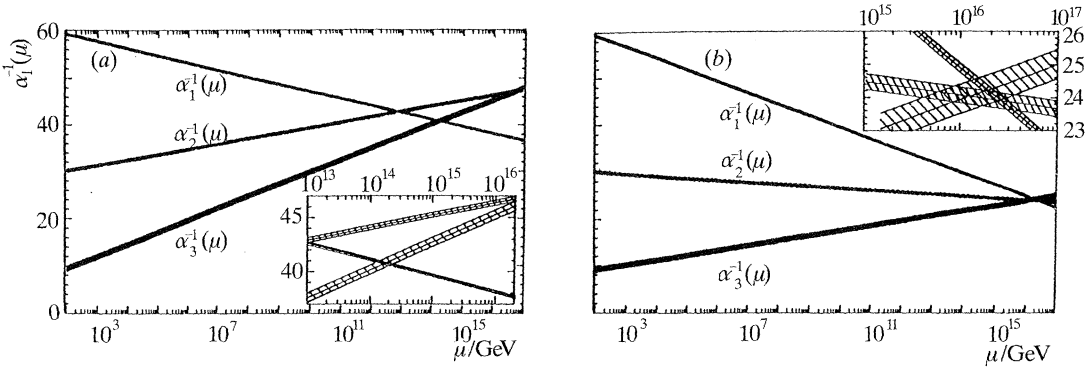

Figure 5. Evolution of the three fundamental coupling constants (*a*) in the minimal SU(5) model containing no new physics above the Z^0 mass, which is clearly excluded by the failure of the three values to coincide at a 'grand unification' mass, and (*b*) in the minimal supersymmetric extension of the model assuming the supersymmetry (SUSY) mass scale to be near the Z^0, in which case the values meet within one standard deviation at a mass above 10^{16} GeV.

of current PS models. However, the effects found in the DELPHI data (Abreu *et al.* 1990*a*; De Angelis 1990; De Angelis & Demaria 1990) are in excellent agreement with the predictions of the PS model.

6. Beyond the Standard Model

If the details of the soft processes may be considered to be well enough understood, one may extend what has been learned in the other direction, to higher energies, to test possible extensions of the Standard Model. The values of $\alpha_s(M_Z)$ and of the other electroweak coupling constants, all of which are now best measured in studies of Z^0 decays at LEP, may be evolved to higher energies using equations such as equation (4). The integers in this equation, which determine the energy evolution, depend on the symmetry and on the consequent fundamental particle content of the theory. In a correct extension of the Standard Model, the values of the effective coupling constants will evolve in such a way that all three values coincide at some very high 'grand unification' mass.

Figure 5, taken from Amaldi *et al.* (1991), shows that in the simple SU(5) model which contains no new physics above the Z^0 mass the values fail to coincide. This model (already excluded by the failure to detect proton decay) is therefore now excluded independently by the LEP data. In addition, many Standard Model extensions not already excluded by other data can, in principle, be excluded in this way. However, in the minimal supersymmetric model (MSSM) the values coincide well. The energy at which the lines change from having the simple SU(5) slopes to having the lower MSSM slopes indicates the typical supersymmetric particle mass. The most favoured value is O(1 TeV). The main contribution to the uncertainty is the uncertainty in $\alpha_s(M_Z)$. Reducing this uncertainty further is clearly an extremely important goal.

7. Conclusion

LEP has confirmed its many advantages for the study of jets: (1) as at PETRA and PEP, the initial state is well defined ($q\bar{q}$) and seems background free, and in addition the distortions of the event shapes by initial state radiation are largely eliminable,

(2) because of the higher energy than at PETRA and PEP, the jets are better separated, the perturbative and hadronization stages are better distinguished, and the lower value of α_s reduces the effect of unknown higher-order corrections, and (3) already each LEP experiment has at least four times more statistics than each PETRA or PEP experiment accumulated, and the statistics should increase again by a factor 5–10 in the coming year. Consequently, LEP is now the main test-bed for QCD, using analyses and techniques adapted and extended from the PETRA and PEP experiments.

In this first full year of LEP operation, many features of the data on Z^0 decays into hadronic final states have been studied. All are in remarkably good agreement with the predictions of Monte Carlo models in which the 'hard' processes and the 'soft' parton showering are described by perturbative QCD calculations. Furthermore, it appears that even the 'very soft' hadronization processes are beginning to be understood analytically in terms of QCD, not only in terms of QCD-inspired but slightly *ad hoc* phenomenological models. The $\alpha_s(M_Z)$ values extracted from detailed comparisons of jet rates, event shapes, and the energy–energy correlation and its asymmetry with second-order ($O(\alpha_s^2)$) QCD matrix element predictions. All are in excellent agreement.

These values of $\alpha_s(M_Z)$ may be combined with those of the electroweak coupling constants, now also best measured in Z^0 decay, to test deeper gauge theories that unify QCD with the electroweak theory. This strongly excludes the so-called 'desert scenario', in which nothing undiscovered lies at energies above the Z^0, and can tightly constrain what might possibly populate the desert. But already the dominant uncertainties in the value of $\alpha_s(M_Z)$ are theoretical in nature. Consequently, further progress in this fundamental direction will require a considerable effort to improve and extend the precision of the existing theoretical QCD calculations.

References

Aarnio, P., *et al.* 1990 Study of hadronic decays of the Z^0 boson. *Phys. Lett.* B **240**, 271–282.

Abreu, P., *et al.* 1990*a* A study of intermittency in hadronic Z^0 decays. *Phys. Lett.* B **247**, 137–147.

Abreu, P., *et al.* 1990*b* A comparison of jet production rates on the Z^0 resonance to perturbative QCD. *Phys. Lett.* B **247**, 167–176.

Abreu, P., *et al.* 1990*c* Energy–energy correlations in hadronic final states from Z^0 decays. *Phys. Lett.* B **252**, 149–158.

Abreu, P., *et al.* 1991*a* Experimental study of the triple gluon vertex. *Phys. Lett.* B **255**, 466–476.

Abreu, P., *et al.* 1991*b* Charged particle multiplicity distributions in Z^0 hadronic decays. Preprint CERN-PPE/90-173. *Z. Phys.* C **50**, 185–194.

Adamus, M., *et al.* 1987 Maximum particle densities in rapidity space of π^+p, K^+p and pp collisions at 250 GeV/*c*. *Phys. Lett.* B **185**, 200–210.

Adeva, B., *et al.* 1990*a* A test of QCD based on 4-jet events from Z^0 decays. *Phys. Lett.* B **248**, 227–234.

Adeva, B., *et al.* 1990*b* Determination of α_s from jet multiplicities measured on the Z^0 resonance. *Phys. Lett.* B **248**, 464–472.

Adeva, B., *et al.* 1991 Determination of α_s from energy–energy correlations measured on the Z^0 resonance. *Phys. Lett.* B **257**, 469–478.

Akrawy, M. Z., *et al.* 1989 A study of jet production rates and a test of QCD on the Z^0 resonance. *Phys. Lett.* B **235**, 389–398.

Akrawy, M. Z., *et al.* 1990*a* A measurement of global event shape distributions in hadronic Z^0 decays. *Z. Phys.* C **47**, 505–522.

Akrawy, M. Z., *et al.* 1990*b* A study of coherence of soft gluons in hadron jets. *Phys. Lett.* B **247**, 617–628.

Akrawy, M. Z., *et al.* 1990*c* A measurement of energy correlations and a determination of $\alpha_s(M_Z)$ in e^+e^- annihilations at $\sqrt{s} = 91$ GeV. *Phys. Lett.* B **252**, 159–169.

Akrawy, M. Z., *et al.* 1991*a* A study of angular correlations in 4-jet final states of hadronic Z^0 decays. *Z. Phys.* C **49**, 49–58.

Akrawy, M. Z., *et al.* 1991*b* A study of the recombination scheme dependence of jet production rates and of $\alpha_s(M_Z)$ in hadronic Z^0 decays. Preprint CERN-PPE/90-143 and erratum. *Z. Phys.* C **49**, 375–384.

Amaldi, U., De Boer, W. & Fürstenau, H. 1991 Comparison of grand unified theories with electroweak and strong coupling constants measured at LEP. *Phys. Lett.* B **260**, 447–455.

Bartel, W., *et al.* 1986 Experimental studies on multijet production in e^+e^- annihilation at PETRA. *Z. Phys.* C **33**, 23–32.

Bialas, A. & Peschanski, R. 1986 Moments of rapidity distributions as a measure of short range fluctuations in high energy collisions. *Nucl. Phys.* B **273**, 703–718.

Bialas, A. & Peschanski, R. 1988 Intermittency in multiparticle production at high energy. *Nucl. Phys.* B **308**, 857–867.

Biddulph, P. & Thompson, G. 1989 Improved modelling of independent parton hadronisation. *Comp. Phys. Commun.* **54**, 13–22.

Braunschweig, W., *et al.* 1989 Study of intermittency in e^+e^- annihilation into hadrons. *Phys. Lett.* B **231**, 548–556.

Burnett, T. H., *et al.* 1983 Extremely high multiplicities in high energy nucleus–nucleus collisions. *Phys. Rev. Lett.* **50**, 2062–2065.

Chlipapnikov, P. & Tchikilev, O. G. 1990 KNO scaling as a property of a stochastic branching process. *Phys. Lett.* B **235**, 347–350.

De Angelis, A. 1990 Study of intermittency in hadronic Z^0 decays. Preprint CERN-PPE/90-129. *Mod. Phys. Lett.* A **5**, 2395–2402.

De Angelis, A. & Demaria, N. 1990 Intermittency in DELPHI, an update. DELPHI note 90-45 PHYS 72.

De Boer, W. 1990 Review of QCD at LEP and SLC. Preprint CERN-PPE/90-161. *Proc. Topical Conference of the 18th SLAC Summer Institute on Particle Physics, July 1990, Stanford.*

De Boer, W., Fürstenau, H. & Köhne, J. H. 1991 Optimised second order QCD versus parton shower models in e^+e^- annihilation. *Z. Phys.* C **49**, 141–150.

Decamp, D., *et al.* 1990 Properties of hadronic events in e^+e^- annihilation at $\sqrt{s} = 91$ Gev. *Phys. Lett.* B **234**, 209–218.

Decamp, D., *et al.* 1991*a* Measurement of the strong coupling constant α_s from global event shape variables of hadronic Z^0 decays. *Phys. Lett.* B **255**, 623–633.

Decamp, D., *et al.* 1991*b* Measurement of α_s from the structure of particle clusters produced in hadronic Z^0 decays. Preprint CERN-PPE/90-196. *Phys. Lett.* B **257**, 479–491.

Dokshitzer, Yu. L., *et al.* 1988 QCD coherence in high energy reactions. *Rev. mod. Phys.* **60**, 373–388.

Komamiya, S., *et al.* 1990 Determination of α_s from a differential jet multiplicity distribution in e^+e^- annihilations at $\sqrt{s} = 29$ and 91 GeV. *Phys. Rev. Lett.* **64**, 987–990.

Kunszt, Z. & Nason, P. 1989 QCD in 'Z Physics at LEP 1', CERN Yellow Report 89-08, pp. 373–453.

Magnoli, N., Nason, P. & Rattazzi, R. 1990 Tests of QCD from jets on the Z^0 peak. *Phys. Lett.* B **252**, 271–281.

Marchesini, G. & Webber, B. R. 1988 Monte Carlo simulation of general hard processes with coherent QCD radiation. *Nucl. Phys.* B **310**, 461–526.

Pennington, M. R. (ed.) 1991 *Proc. Durham Jet Studies Workshop, December 1990.* (In the press.)

Sjöstrand, T. 1982 The Lund Monte Carlo program for jet fragmentation. *Comp. Phys. Commun.* **27**, 243–284.

Sjöstrand, T. 1983 The Lund Monte Carlo program for e^+e^- jet physics. *Comp. Phys. Commun.* **28**, 229–254.

Sjöstrand, T. & Bengtsson, M. 1987 The Lund Monte Carlo program for jet fragmentation and e^+e^- physics – JETSET version 6.3 – an update. *Comp. Phys. Commun.* **43**, 367–380.